$Z=\sqrt{\left(2πfm-\frac{k}{2πf}\right) ^{2}}+b\^2$

Where

Z = impedance

f = frequency

m = effective mass

k = effective stiffness

b = effective damping

Strictly speaking, these are quantities that must be measured under dynamic (oscillatory) conditions — m is not simply the static mass of the soundboard and bridge; ditto for k and b which is why these are stated as effective quantities. For our purposes the locations of these quantities within the equation is all we need to know so as to form a conceptual sense of how impedance is a function of f, m, k and b.

$Z=\sqrt{\left(fm-\frac{k}{f}\right) ^{2}}+b\^2$

Taking the 2π values out for simplicity, we can find direct and indirect relations to impedance Z. Note that m, k and b are numerators and therefore are directly proportional to Z ----- i.e. any increase to mass or stiffness or damping will increase impedance. Stiffness obtains not only from the physical quantities of ribs, panel, bridges and entire support structures, but also from downbearing. Increases to bearing increase impedance. And all increases to impedance (which impede the input of string energy) encourage sustain (not necessarily good tonal sustain) while discouraging belly compliance and power output.

If we imagine a soundboard made of rigid cast iron, the impedance would be virtually infinite ∞ and the string would vibrate (sustain) a very long time while uselessly dissipating its energy from a piano tonal standpoint. At the other end of the extremes imagine a soundboard made of thin membrane such as a drum head (think banjo). Such a soundboard would be extremely compliant and would welcome the string energy and dump it almost instantaneously in a power dump of loud attack with no useful sustain.

Also there is a frequency f

$f=\frac{1}{2π} \sqrt{k}/m $

Again, for simplicity we remove the 1/2π and we have

$f= \sqrt{k}/m$

at which mass and stiffness effects cancel each other, and the only contributor to the impedance is damping factor b. At this point Z has achieved a minimum, and vibrational energy at this frequency will be dissipated at the fastest possible rate.

There is much more to this study, but it only gets crazier from here.

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