**An Aural Test for the Correct Estimate of an Equal-Tempered Major Third, as Used in Setting a Stack of Contiguous Major Thirds.**

1. Tune A4>A3>A2
2. Tune A2>C#3, estimate at 4.3 bps
3. Tune C#3>C#4

Here’s the test:

1. Tune C#3>F3 such that A2-C#3 and C#3-F3 are equal-beating.
2. If F3-A3 are now-equal beating with A3-C#4, the C#s are correct for equal temperament, they are tempered at one third of the *lesser diesis*.

Proof:

1. Contiguous equal-tempered Major Thirds beat at the ratio of 4:5, lower to upper third, each tempered by one-third of the *diesis*.
2. If I lower F3 to create equal-beating thirds A2-C#3-F3 (4:4) I have effectively removed one fifth of the one-third *diesis* from C#3-F3 and transferred it to F3-A3.

Since F3-A3-C#4, correctly tuned beat at 4:5, transferring one-fifth of the one-third *diesis* produces equal-beating contiguous thirds, changing the lower ratio from 4:5 to 4:4 and the upper pair ratio from 4:5 to 5:5.

Sequentially, the four thirds should beat 4:4:8:8

1. If the C#s are too flat, the percentage of *diesis* to be divided by F3 will be too great, and one or both of the C#3-F3-A3 thirds will beat too fast relative to the too slow A-C# thirds.
2. If the C#s are too sharp, the percentage of *diesis* to be divided by F3 will be too small, and one or both of the C#3-F3-A3 thirds will beat too slow relative to the too fast A-C# thirds.
3. Thus, assuming correct octaves, the only division of a Major Tenth that will produce two pairs of equal-beating lower and upper contiguous Major Thirds requires that the lowest and highest Major Thirds are tempered to exactly one-third of the *diesis*. [This would also be true, for example, of F3-A3-C#4-F4-A4.]

Ed Sutton 5/6/2019